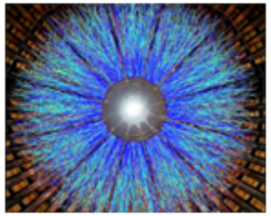
School of Collective Dynamics in High Energy Collisions
June 7 - 11, 2010



Toward the QCD Equation of State

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Basics I

Hydrodynamics, a warm-up

Equations of motion of relativistic hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0 \Leftrightarrow$$

local conservation of energy and momentum

the energy-momentum tensor

and

$$\partial_\mu j_i^\mu = 0 \Leftrightarrow$$

conservation of charge *i* current

the charge current

charges = baryon number, electric charge, strangeness ...

Ideal Fluid

Assumption of an "ideal" (non-dissipative) fluid reduces computational effort:

$$\boxed{\begin{aligned} T^{\mu\nu} &= (\epsilon + p)\, u^\mu u^\nu - p g^{\mu\nu} \\ j_i^\mu &= n_i u^\mu \end{aligned}}$$

$$\left.\begin{array}{ll} \epsilon & \text{energy density} \\ p & \text{pressure density} \\ n_i & \text{number density for charge } i \\ u^\mu & \text{flow 4-velocity} \end{array}\right\} \text{in the local restframe}$$

The 4-dimensional flow pattern is a 4-velocity field driven by the gradients of pressure and energy densities

In order to calculate the $n_i(T)$, it is important to note that local kinetic and chemical equilibrium are implied.
Then, n_i is calculated from the Partition Function $\ln Z_i$

To solve the above equations, one needs the relation between p and ϵ ,
the so-called EQUATION OF STATE (EOS)

Hydro in A+A collisions

To treat A+A collision dynamics as a hydrodynamical flow process, one has to:

Initialize the flow field (for primordial geometry and dynamics) at early CM time after interpenetration: $\tau \approx 2R/\gamma \leq 0.2\mathrm{fm}/c$ at RHIC

↓

Finalize or freeze-out the emerging particles at late times (not our subject here)

Basics II

Equation of State (EoS): $p(\epsilon)$

The EoS "inhales the physics" into hydrodynamics

Remember High School: Ideal one-atomic gas

$$pV = NkT$$

$$p = nkT$$ n number density $\frac{N}{V}$

nonrelativistic limit: $\bar{E}_{\rm kin} = \frac{3}{2}kT$

$$n\bar{E} = \epsilon$$ the energy density

$\boxed{p = \frac{2}{3}\epsilon \quad \text{classical ideal gas EoS, relativistic:} \quad p = \frac{1}{3}\epsilon}$

$\epsilon = \mathrm{const} \cdot T^4$ Stefan Boltzmann

QCD Toy Model EoS

A "simple" gas of $\pi^{+,-,0}$ and of quarks (2 flavours) and gluons:

$$p_\pi = 3\frac{\pi^2}{90}T^4$$ for 3 pion charges and massless pions

$$p_{qg} = \{\underbrace{2 \cdot 8}_{\substack{\text{spin x color} \\ \text{gluons}}} + \frac{7}{8}(\underbrace{3 \cdot 2 \cdot 2 \cdot 2}_{\substack{\text{3 col x 2 flav x 2 spin x 2 part/antipart} \\ \text{quarks}}})\}\frac{\pi^2}{90}T^4 - B(T)$$

B is the *"bag pressure"* which expresses the difference of vacuum and in-medium QCD pressure

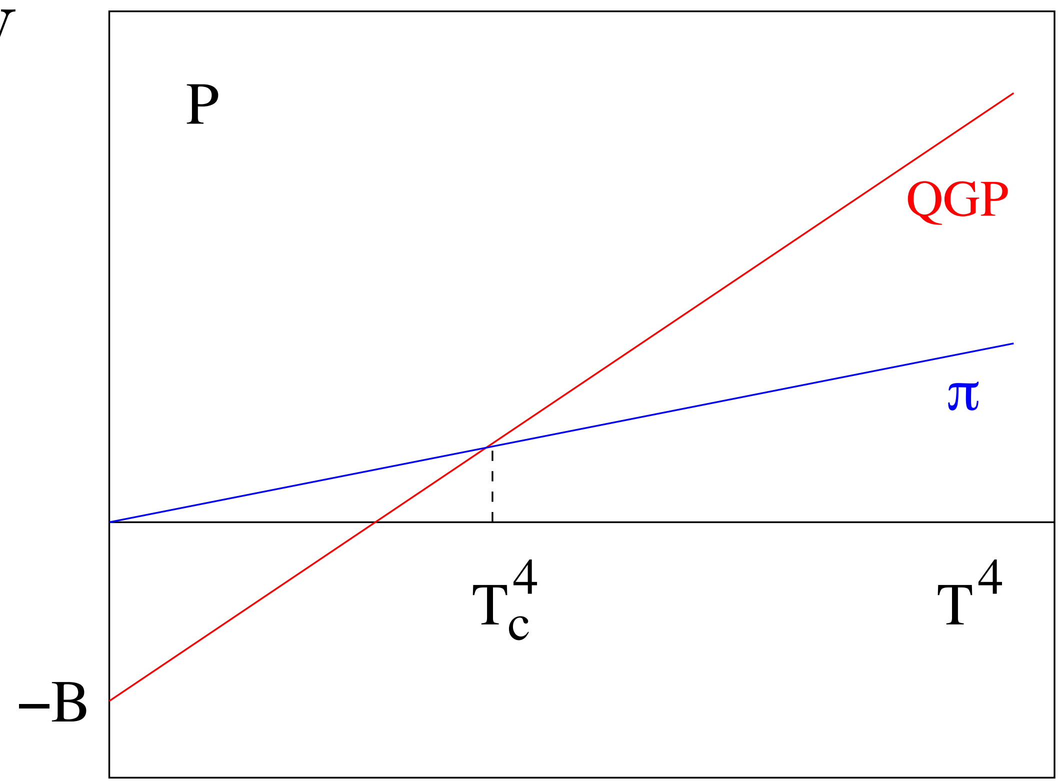
Result:

- A crossing at $T_C \approx 150\mathrm{MeV}$ first order phase transition

- The stable phase is the one with the higher pressure

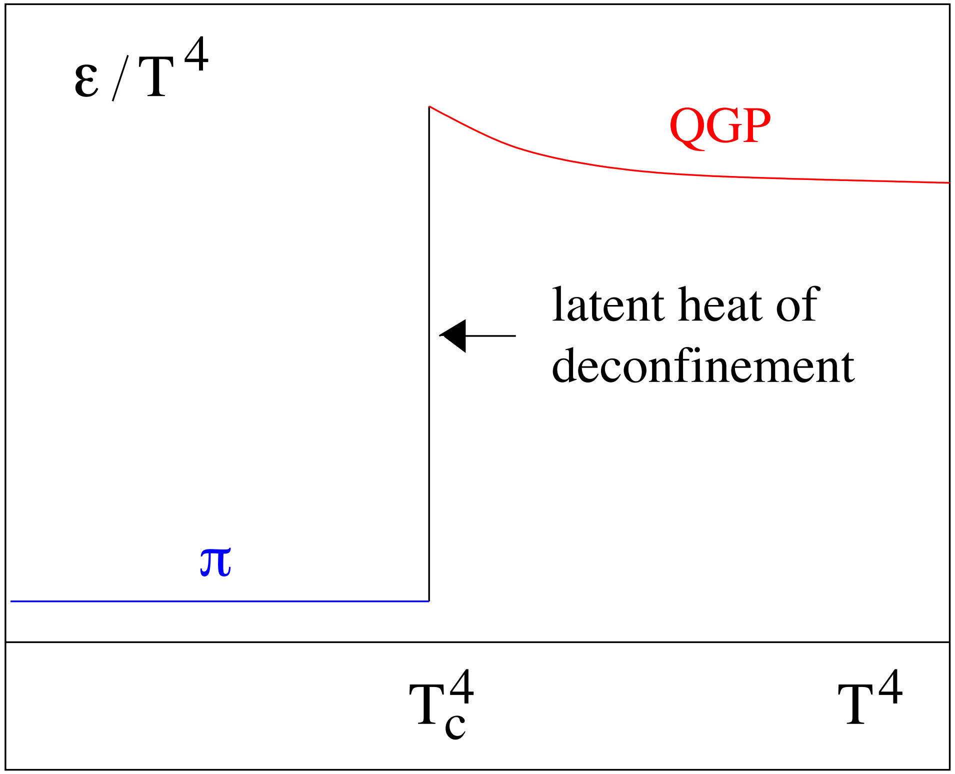
- Resulting "QCD" EoS:

$$\epsilon - 3p = 4B(T)$$



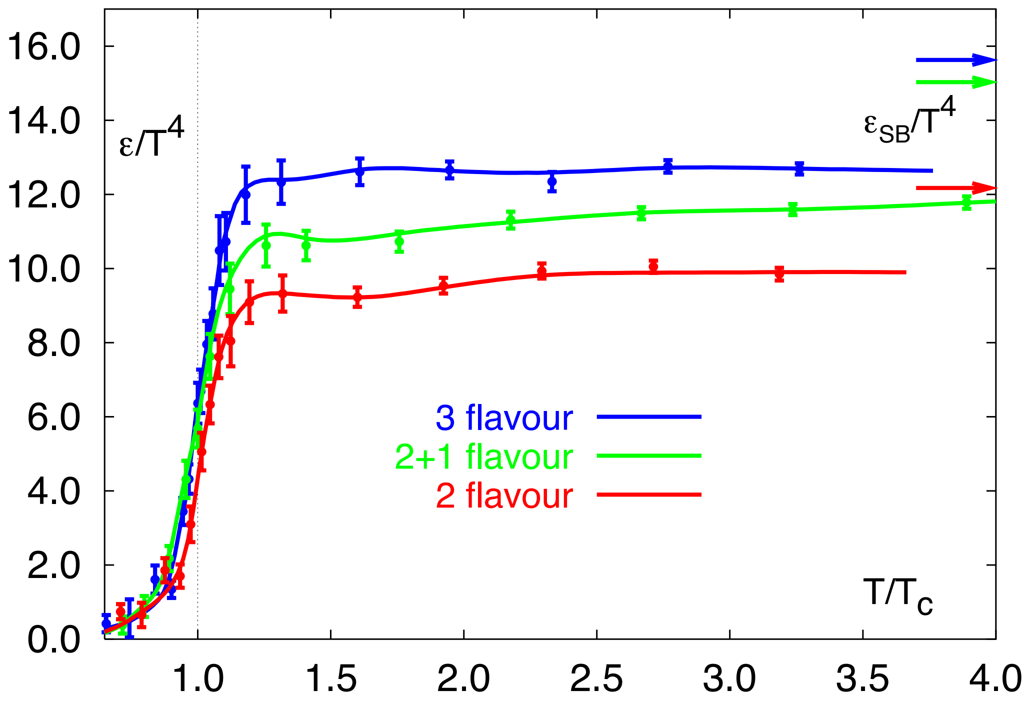
Result:

- Two phases separated by the latent heat jump at T_C
- $\Delta\epsilon = \epsilon_{\mathrm{QGP}}(T_C) - \epsilon_\pi(T_C)$
  $= 4B$

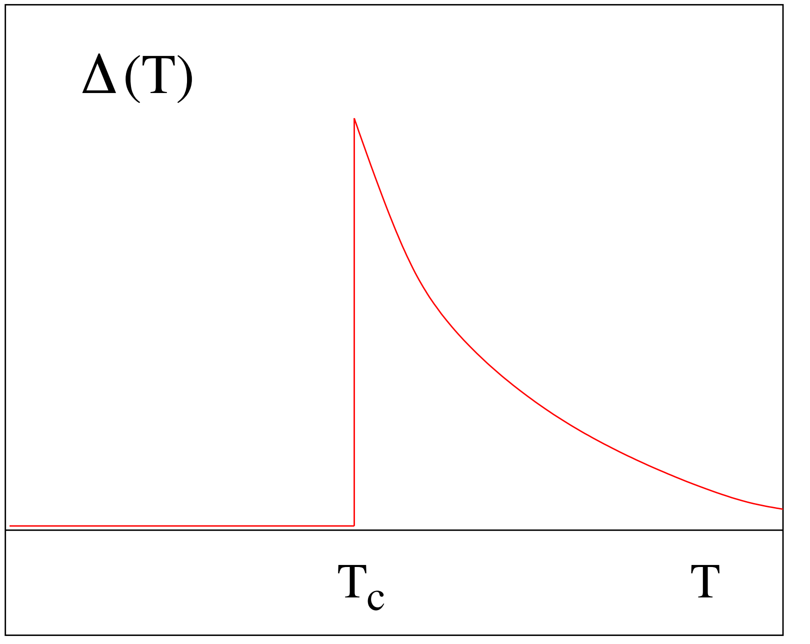


And, in fact, lattice QCD shows this!

Lattice QCD



Energy density



Trace anomaly

$$\epsilon - 3p \neq 0$$

QCD trace anomaly implies massive partons in non-perturbative vacuum

Effect disappears at $T \geq 3T_C$

The term “trace anomaly”: $\mathrm{tr}(T^{\mu\nu}) = \epsilon - 3p = 0$ in ideal massless gas

Application to A+A collisions

- A hydrodynamic expansion evolution in a A+A collision follows a bundle of trajectories in $\{\epsilon, p, n_i\}$ space

- At $T > T_C$, one will employ a QGP EoS, starting at the end of initialization time

- $\boxed{\text{At RHIC top energy, this time may be as “early” as } 0.5\mathrm{fm}/c}$

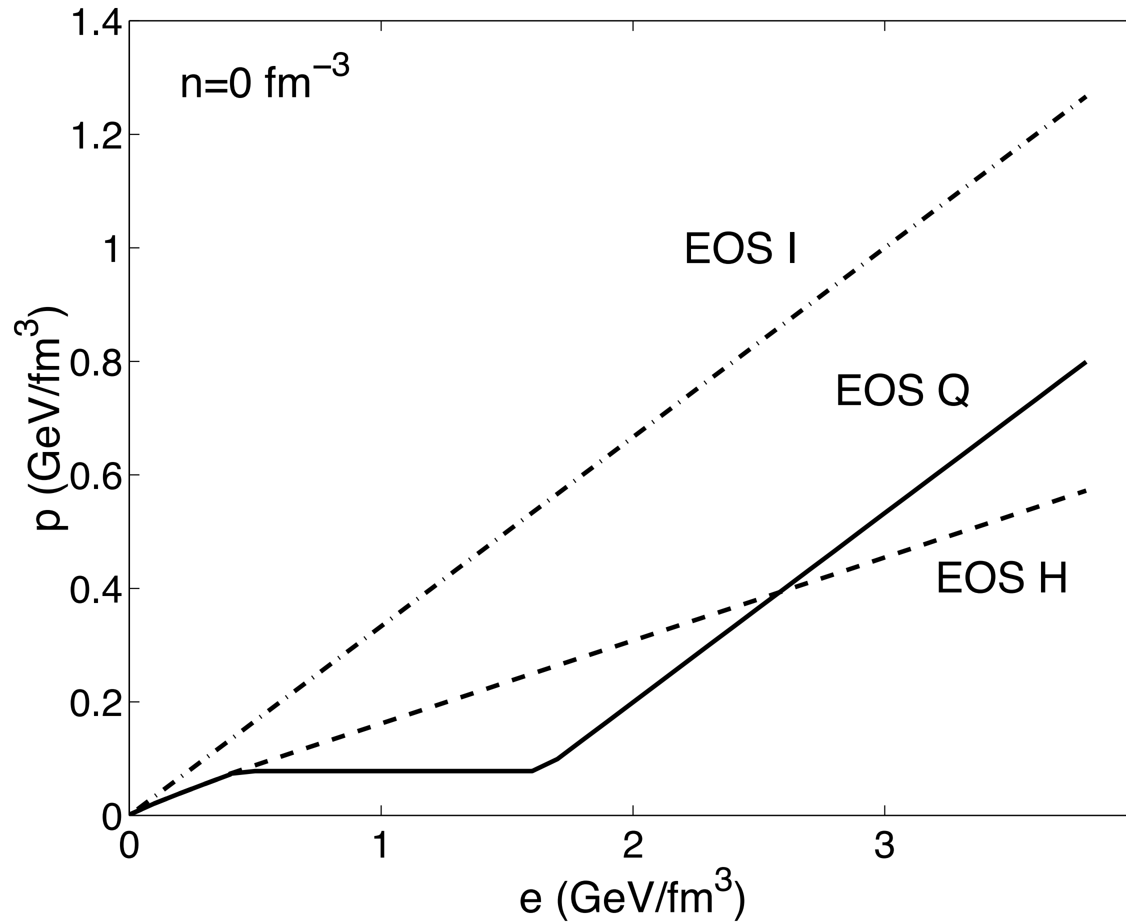
- As ϵ falls below about $1\mathrm{GeV}/\mathrm{fm}^3$
  → switch to hadronic, grand-canonical EoS
  or
  → terminate hydro expansion (Cooper-Frye), match to hadron-resonance transport

A trial EoS

Illustrates EoS matching between QGP and hadron resonance gas (Kolb, Sollfrank, Heinz PRC 62:054909,2000)

Strategy:

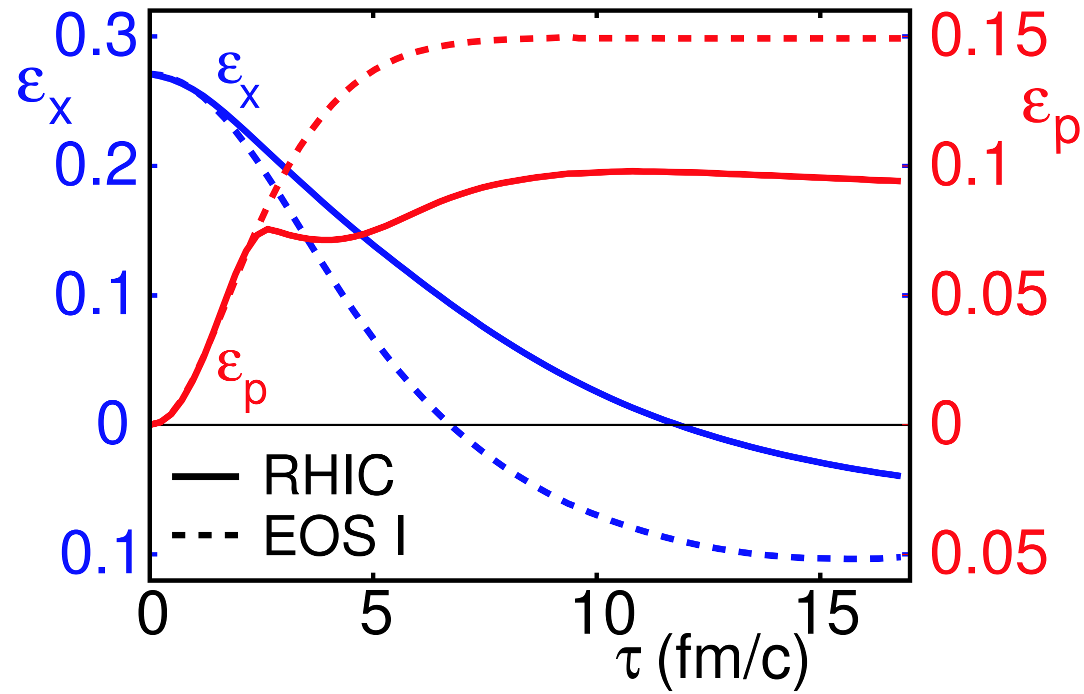
- Outline EoS-sensitve observables
- try out alternative EoS's
- unfortunately (?) involving different phase transition models



The elliptic flow anisotropy signal

Kolb and Heinz in QGP 3 (Hwa, ed.)

Time evolution in relativistic hydro:
Initial space eccentricity ϵ_x in A+A collisions
vs.
generation of a momentum space anisotropy ϵ_p
Measured by elliptic flow v_2



Flow established at $\tau \leq 2\mathrm{fm}/c$ at top RHIC energy

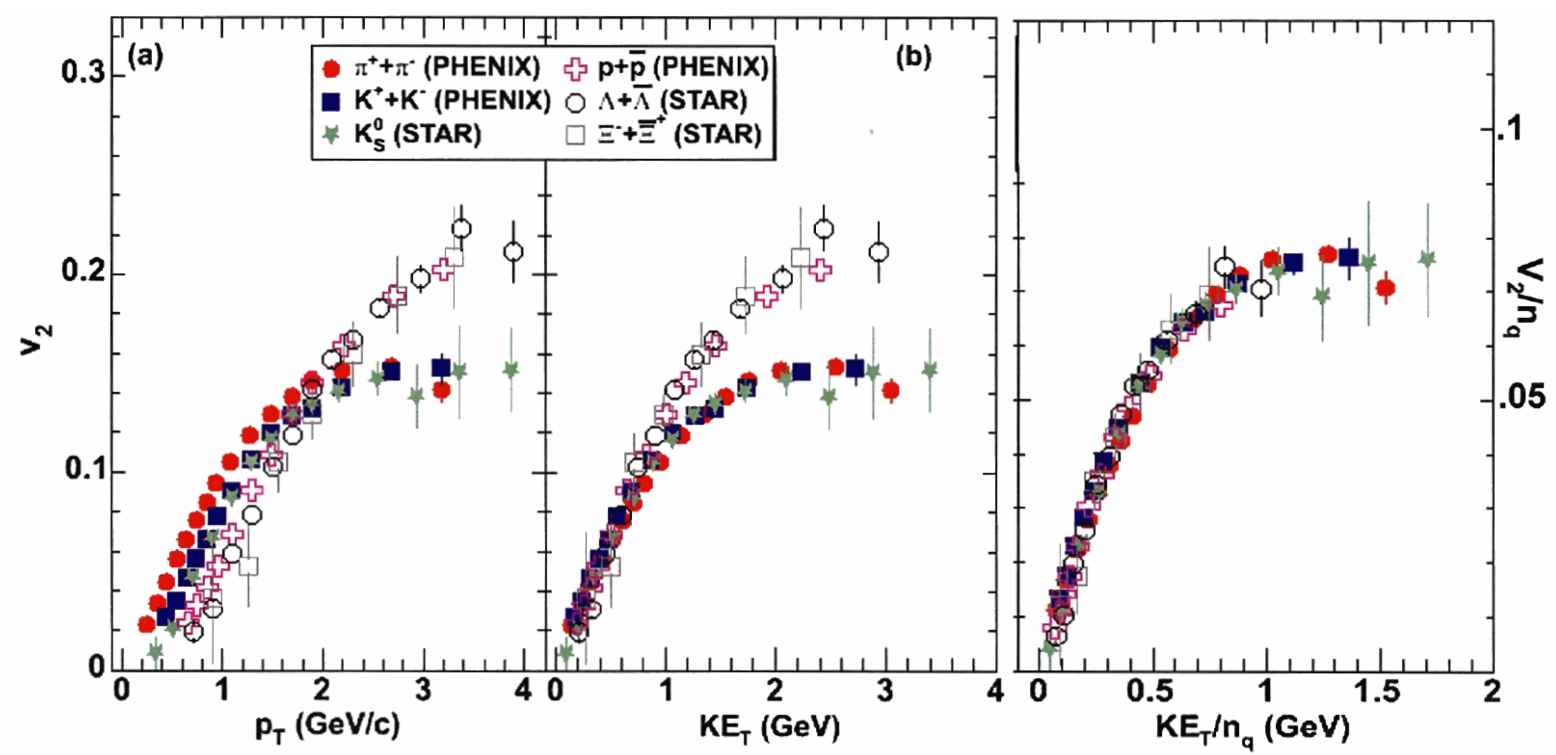
Primordial QGP signal!

Elliptic Flow

A QGP signal

R.A.Lacey and A.Taranenko: PoSCFRNC2006:021,2006

Elliptic flow scales with quark number!



Sensitivity to the partonic EoS